WorkBook

SURFACE AREA

Using a method and/or a formula

WorkNotes

WorkBook

SURFACE AREA OF SOLIDS

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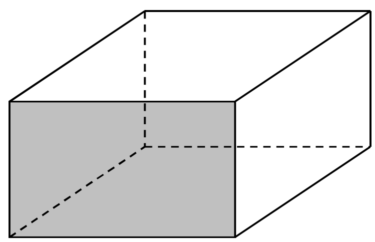
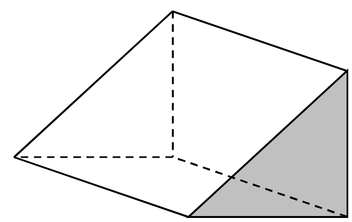
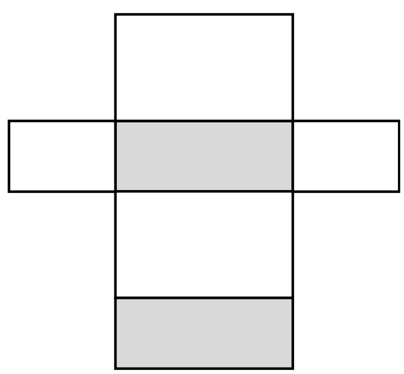
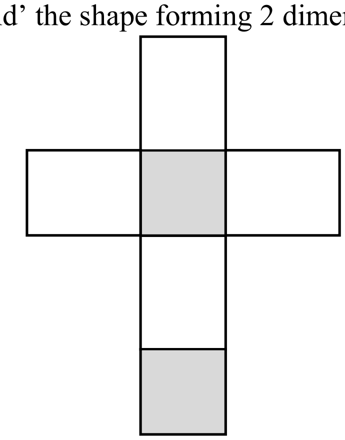
Solids	• The surface area of any solid is the sum of the areas of all its faces. The areas of the faces are either;
	  ○ given, or
	  ○ have to be calculated.
	• The surface area of some solids is calculated using a formula.
	• It will always simplify the calculation of the surface area if you describe the faces as the first step in calculating the surface area.

Prisms	• A prism is a polyhedron with congruent polygonal cross-sections from the base to the 'top' joined by multiple rectangular faces. All cross-sections parallel to the base are translations of the base. The rectangular faces are perpendicular to the base unless it is an <u>oblique prism</u> (slanted) which have the base and top of the prism joined by parallelograms and/or rectangles. • A prism is usually named by the shape of its base. A cube is an exception. • The surface area is calculated by adding the area of all the faces. • Always describe the faces as the first step in calculating the surface area. **Examples**  Rectangular Prism 3 pairs of rectangles  Triangular Prism A pair of triangles and 3rectangles • The **net** of a solid is what you get if you 'unfold' the shape forming 2 dimensions.  **Rectangular prism** The surface is 3 pairs of rectangles  **Square prism** The surface is a pair of squares and a set of 4 rectangles

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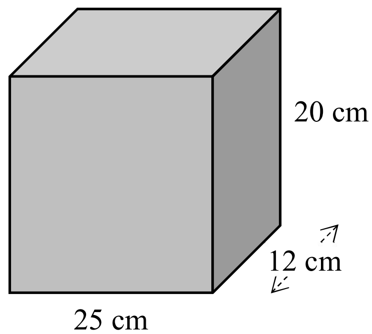
<table>
<tr>
<td>Rectangular and Square Prisms</td>
<td>

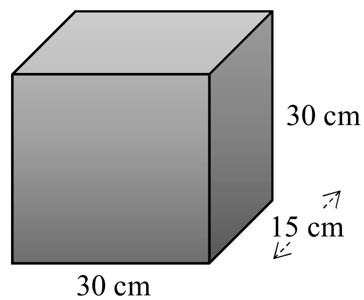
- The cross section is a rectangle or a square.
- Surface area is calculated by adding the areas of each of the faces
- The faces that form the prism are are flat or planar surfaces. The faces are joined at **edges**.

Eg
Find the surface area of the following box.



The surface is 3 pairs of rectangles
$SA = 2 \times (25 \times 12) + 2 \times (20 \times 12) + 2 \times (25 \times 20)$
$= 600 + 480 + 1000$
$= 2080$
$\therefore$ Surface area is 2080 cm^2

Eg
Find the surface area of the following square prism.



The surface is a pair of squares and a set of 4 rectangles
$SA = 2 \times 30^2 + 4 \times (30 \times 15)$
$= 1800 + 1800$
$= 3600$
$\therefore$ Surface area is 3600 cm^2

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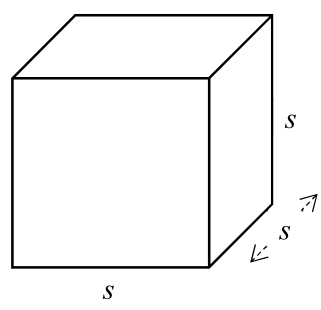
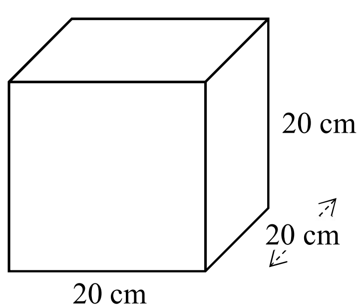
<tr>
<td>Cubes</td>
<td>

- All faces are squares.
- opposite angles are equal.
- The surface area of a cube is calculated multiplying the area of the square faces by 6. Or $\boldsymbol{SA = 6s^2}$, where $\boldsymbol{s}$ is the length of the sides of the square.



Eg
Find the area of the following cube.



$SA = 6s^2$
$= 6 \times 20^2$
$= 2400$
$\therefore$ Surface area is 2400 cm^2

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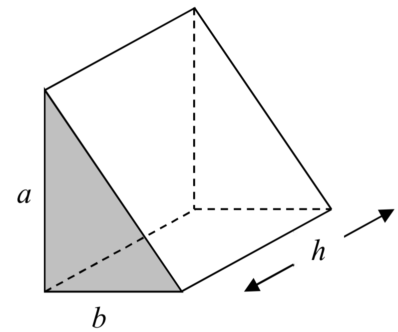
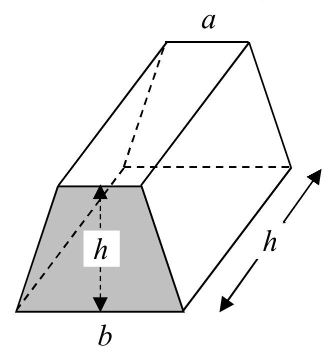
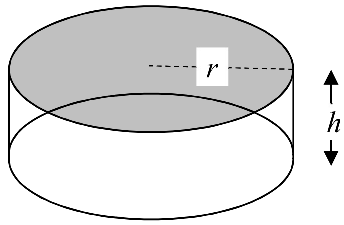
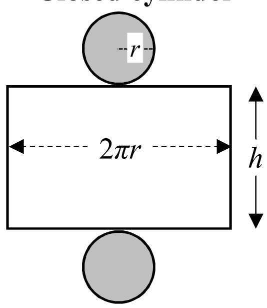
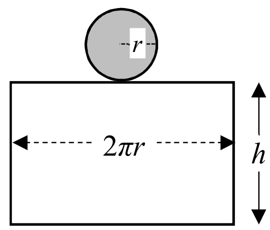
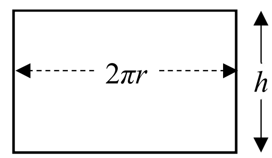
Other Polygonal Bases	• Always describe the faces as the first step in calculating the surface area. • The surface area is calculated <u>adding the</u> <u>area of all the faces</u>.  A triangular prism – a pair of triangles and 3 rectangles  A trapezoidal prism – a pair of trapeziums and 4 rectangles

Cylinders

- A cylinder is a solid with a circular base.
- While it is not a prism it is similar.
- If you deconstruct a cylinder the face joining the circular ends is a rectangle. Try it for yourself. Take a rectangular piece of paper and join the vertical edges without folding the piece of paper. You will make an open cylinder.
- Hence the length of the rectangular face is the circumference of the circle. ie. $\mathbf{2\pi r}$.
- The area of the circles is $A = \pi r^2$ and the area of the rectangle is $A = 2\pi r \times h$.

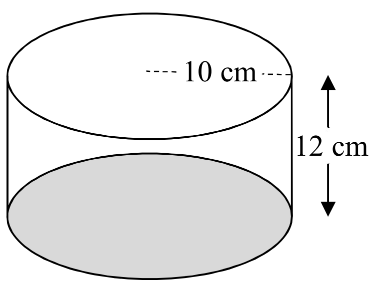


- There are three types of cylinders to consider when calculating surface area.

Closed cylinder	**Open one end cylinder**	**Open cylinder**
		
The surface is a pair of circles and a rectangle	The surface is a circle and a rectangle	The surface a rectangle
Examples Can of baked beans Fuel can	**Examples** Vase Plant pot	**Example** Length of pipe

Eg

Find the surface area of the following closed cylinder correct to 1 decimal place.



SA is a pair of circles and a rectangle

$SA = 2 \times \pi r^2 + 2\pi rh$

$= 2 \times \pi \times 10^2 + 2 \times \pi \times 10 \times 12$

$= 200\pi + 120\pi$

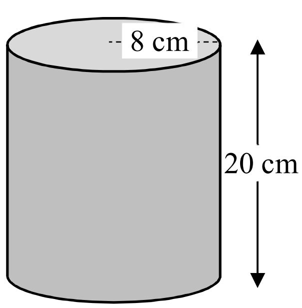
$= 320\pi$

$= 1005.309649148733\ldots$

$\therefore$ Surface area is 1005.3 cm^2

Eg

Find the surface area of the following cylindrical pot, open at the top, correct to the nearest whole number.



SA = A is a circle and a rectangle

$SA = \pi r^2 + 2\pi rh$

$= \pi \times 8^2 + 2 \times \pi \times 8 \times 20$

$= 64\pi + 320\pi$

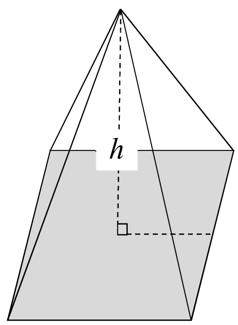
$= 384\pi$

$= 1206.3715789784806\ldots$

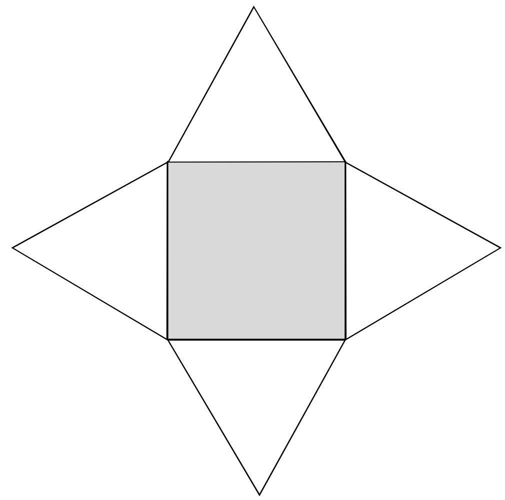
$\therefore$ Surface area is 1206cm^2

Pyramids

- A pyramid has a polygonal (straight sided) base. All other faces are triangles meeting at the apex.
- The triangles in a right pyramid, that is, where the apex is perpendicular above the centre of the base, are isosceles and/or equilateral triangles.
- The surface area is the area of the base plus the area of the triangles.
  - The surface area is the area of a square pyramid is a square plus 4 triangles. The triangles are congruent if the apex is perpendicular above the centre of the square.
  - The surface area is the area of a rectangular pyramid is a rectangle plus 2 pairs of triangles.
  - The surface area is the area of a triangular pyramid is 4 triangles.



A pyramid

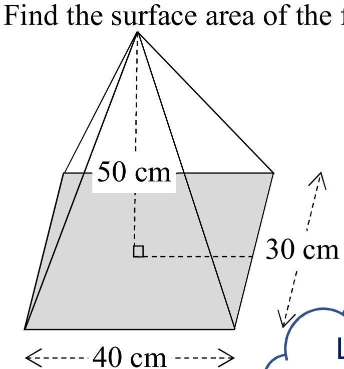


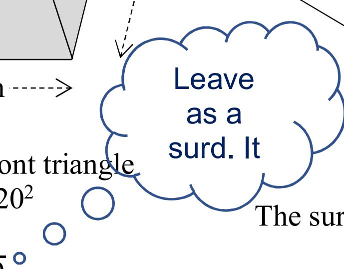
The net of a pyramid

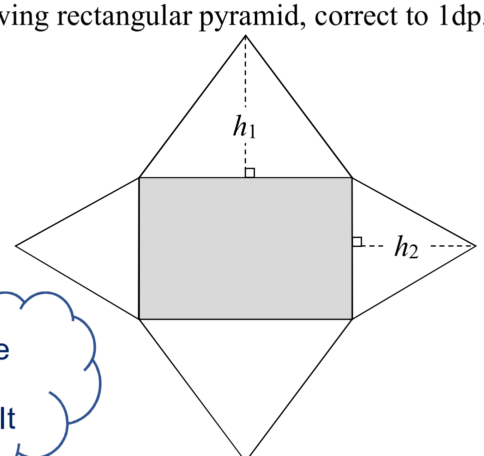
- Calculating the surface area of a pyramid can be a little complex depending on the data.
- If the **height of a pyramid** is given you must calculate the **slant height** of the pyramid. This will give you the height of the triangles which is required to calculate the area of the triangles. Use Pythagoras' Rule to find the slant height.

Eg

Find the surface area of the following rectangular pyramid, correct to 1dp.







Height of front triangle

$h_1^2 = 50^2 + 20^2$

$= 2900$

$h_1 = \sqrt{2900}$

Height of side triangles

$h_2^2 = 50^2 + 15^2$

$= 2725$

$h_2 = \sqrt{2725}$

The surface is a rectangle and 2 pairs of triangles

$SA = (40\times30) + 2\times(\frac{1}{2}\times40\times\sqrt{2900}) + 2\times(\frac{1}{2}\times30\times\sqrt{2725})$

$= 1200 + (40\times\sqrt{2900}) + (30\times\sqrt{2725})$

$= 4920.1118991903841\ldots$

$\therefore$ Surface area is 4920.1 cm^2

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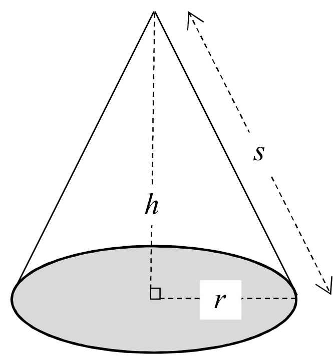
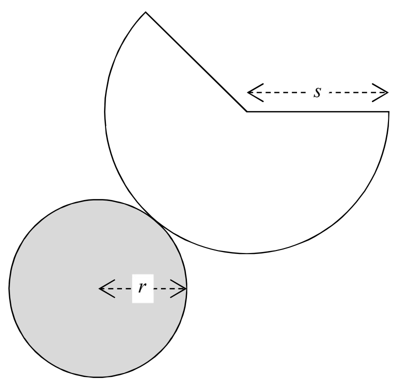
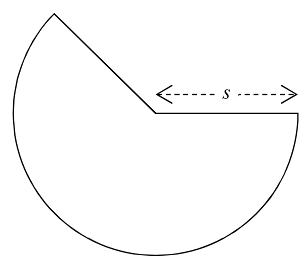
Cones

- A cone is a 3 dimensional object with a circular base and line segments meeting at an apex. Alternatively a cone can be seen as being formed by layers of circles with diminishing radii as you move away from the base.
- While a cone is not a pyramid, some definitions of a cone leads to a pyramid being a cone with a polygonal base.
- The **slant height** of a cone is the length from the circumference of the base circle to the apex.
- The **net** of a cone is either;
  - **Closed cone**: A circle and a sector of a circle with an arc length equal to the circumference of the base circle. ie. $\boldsymbol{SA = \pi r^2 + \pi rs}$ where $\boldsymbol{s}$ is the slant height.
  - **Open cone**: A sector of a circle with an arc length equal to the circumference of the base circle. ie. $\boldsymbol{SA = \pi rs}$



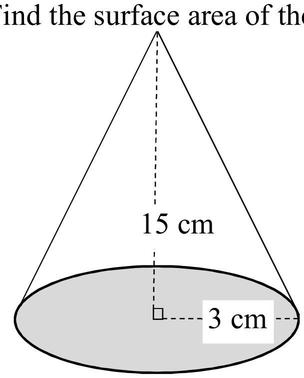


The net of a closed cone



The net of an open cone

Eg
Find the surface area of the following closed cone, correct to 1dp.



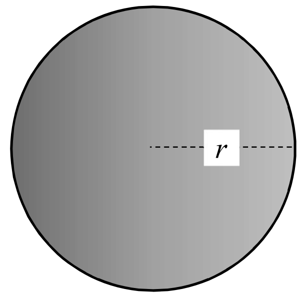
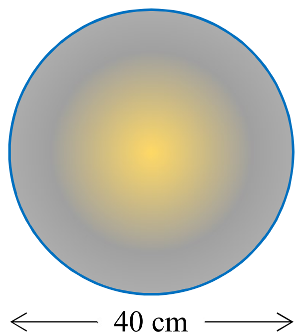
The surface area is a circle and a sector of a circle

$$
\begin{aligned}
SA &= \boldsymbol{\pi r^2 + \pi rs} \\
&= \pi \times 3^2 + \pi \times 3 \times \sqrt{234} \\
&= 9\pi + 45.891175\pi \\
&= 172.4457140820349\ldots
\end{aligned}
$$

$\therefore$ Surface area is 172.4 cm^2

Slant height

$$
\begin{aligned}
s^2 &= 3^2 + 15^2 \\
&= 234 \\
s &= \sqrt{234}
\end{aligned}
$$

Spheres	• A sphere is a 'ball' shape. • A sphere has only one face. • The surface area of a sphere is calculated using the formula $\boldsymbol{A = 4\pi r^2}$.  Eg Find the surface area of the following 40 cm diameter beach ball, correct to the nearest whole number.  **Note:** Since the diameter is given use $r = 40 \div 2$ which is 20 cm $SA = \boldsymbol{4\pi r^2}$ $= 4 \times \pi \times 20^2$ $= 1600\pi$ $= 5026.548245743669\ldots$ $\therefore$ Area is 5027 cm^2