WorkBook

ORDER OF OPERATIONS

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Purpose
A consistent order of operations is very important in guaranteeing the calculation of the single correct solution to an arithmetic problem involving different operations.

Terms
The terms used are usually introduced in stages. Don't be overly concerned about the terms not used in your stage of learning.

The terms include;

Operators	
Addition	+
Subtraction	–
Multiplication	×
Division	÷
Powers & Radicals	
Indices (Exponents or powers)	x^2, x^3, $x^{½}$etc. where *x* is any number
Roots	$\sqrt{x}$, $\sqrt[3]{x}$, etc. where *x* is any number. Note: $x^{½}$ is $\sqrt{x}$, etc
Grouping symbols	
parentheses	( )
brackets	[ ]
braces	{ }
radicals	$\sqrt{x-2}$, $\sqrt[3]{2-x}$, etc.
fraction lines	$\frac{3+5}{5-3}$, etc

Important information
These concepts are the basis of the order of operations.

- Subtraction is the inverse operation of addition. An inverse operation is an operation that undoes what was done by the other operation.
- Division is the inverse operation of multiplication.
- Roots are the inverse operation of powers.
- Multiplication is a 'shortcut' for addition. Eg. 4 × 2 is 2 + 2 + 2 + 2
- Powers (indices or exponents) are a 'shortcut' for multiplication. Eg. 10^4 is 10 × 10 × 10 × 10

Order of operations
This list of simple examples demonstrates the order of operations. There is a summary at the end of the examples.

Addition and subtraction		Calulations only involving addition and subtraction then calculate left to right.
1	4 + 7 – 2 + 1 = 10	You add 4 and 7 to get 11, then subtract 2 to get 9, then add 1 to get 10.
2	8 – 4 – 2 + 5 = 7	You subtract 4 from to get 4, then subtract 2 to get 2, then add 5 to get 7.

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Multiplication and division		Calulations only involving multiplication and division then calculate left to right.
3	$4 \times 6 \div 2$ $= 12$	You multiply 4 by 6 to get 24, then divide by 2 to get 12.
4	$24 \div 3 \times 2 \div 4$ $= 4$	You divide 24 by 3 to get 8, then multiply by 2 to get 16 the divide by 4 to get 4.
All operators		For calulations involving mixes of addition/subtraction and multiplication/division you **muliply/divide first <u>then</u> add/subtract**. Note: Steps are shown
5	$4 \times 6 + 2$ $= 24 + 2$ $= 26$	You multiply 4 by 6 to get 24, then divide by 2 to get 12.
6	$4 + 6 \times 2$ $= 4 + 12$ $= 16$	You multiply 6 by 2 to get 12, then add 4 to get 16.
7	$12 + 6 \div 2$ $= 12 + 3$ $= 15$	You multiply 6 by 2 to get 3, then add 12 to get 15.
8	$12 \div 6 + 2$ $= 2 + 2$ $= 4$	You multiply 12 by 6 to get 2, then add 2 to get 4.
9	$14 \times 5 - 2$ $= 70 - 2$ $= 68$	You multiply 14 by 5 to get 70, then subtract 2 to get 68.
10	$14 - 5 \times 2$ $= 14 - 10$ $= 4$	You multiply 5 by 2 to get 10, then subtract the 10 from 14 to get 4.
11	$24 \div 8 - 2$ $= 3 - 2$ $= 1$	You divide 24 by 8 to get 3, then subtract 2 to get 1.
12	$24 - 8 \div 2$ $= 24 - 4$ $= 20$	You divide 8 by 2 to get 4, then subtract the 4 from 24 to get 20.
Grouping symbols		For calulations involving grouping symbols you **calculate the operation(s) within the grouping symbols <u>then</u> complete other operations**.
13	$(4 + 6) \times 2$ $= 10 \times 2$ $= 20$	4 + 6 is in the grouping symbols so you add 4 and 6 to get 10, then multiply by 2 to get 20.
14	$(12 + 6) \div 2$ $= 18 \div 2$ $= 9$	12 + 6 is in the grouping symbols so you add 12 and 6 to get 18, then divide by 2 to get 9.
15	$4 \times (6 + 2)$ $= 4 \times 8$ $= 32$	6 + 2 is in the grouping symbols so you add 6 and 2 to get 8, then multiply by 4 to get 32.
16	$12 \div (6 - 2)$ $= 12 \div 4$ $= 3$	6 – 2 is in the grouping symbols so you subtract 2 from 6 to get 4, then divide by 12 by 4 to get 3.

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Powers (indices or exponents)		Calulations only involving powers (indices or exponents) are treated the same as grouping symbols, that is, **powers are done <u>before</u> multiplication/division and addition/subtraction**.
17	$4 + 6 \times 3^2$ $= 4 + 6 \times 9$ $= 4 + 54$ $= 58$	The 3 has a power of 2, so 3 squared is 9, then multiply the 9 by 6 to get 54 and finally you add 4 to get 58.
18	$4^2 + 6 \times 3$ $= 16 + 6 \times 3$ $= 16 + 18$ $= 34$	The 4 has a power of 2, so 4 squared is 16, then multiply the 6 by 3 to get 18 and finally you add the 16 and 18 to get 34.
19	$4 \times (6 - 3)^2$ $= 4 \times 3^2$ $= 4 \times 9$ $= 36$	6 – 3 is in the grouping symbols so you subtract 3 from 6 to get 3, then because the answer inside of the grouping symbols is squared, you square the 3 to get 9, and finally, multiply by 4 to get 36.
20	$4^2 \times (6 - 3)$ $= 4^2 \times 3$ $= 16 \times 3$ $= 48$	6 – 3 is in the grouping symbols so you subtract 3 from 6 to get 3, then because the 4 is squared, you square the 4 to get 16, and finally, multiply the 16 and the 3 to get 48.

Summary
In basic calculations the order of operations is a simple process that must be followed to get the one correct solution. However, when the calculation is more complex, care must be taken to follow the rules.

The steps for the Order of Operations

<u>First</u> calculate operations within grouping symbols
<u>then</u> calculate any powers not already calculated
<u>then</u> calculate any multiplications/divisions
<u>then</u> calculate any and additions/subtractions.

ACRONYMS/MNEMONICS
There are several memory devices to assist in remembering the order of operations. They vary depending on location. They all have their weaknesses unless their meaning is fully understood and are meant to aid your memory, and remember, <u>they are not the rule</u>.

Three of them are;
BODMAS – **B**rackets **O**ver **D**ivisions/ **M**ultiplications then **A**ddition/**S**ubtraction
BIDMAS – **B**rackets then **I**ndices then **D**ivisions/ **M**ultiplications then **A**ddition/**S**ubtraction
PEDMAS – **P**arentheses then **E**xponents then **D**ivisions/ **M**ultiplications then **A**ddition/**S**ubtraction

Note: A lot of individuals have their own word for the '**O**' in **BODMAS**, such as 'of' and 'order', however, the above version is what appears in the early NSW textbooks and is more meaningful than some of the modern versions. **BODMAS** is the most common acronym used in Australian schools.

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Advanced Order of Operations

Below are some examples of order of operations that include the 4 operations, grouping (including parentheses, brackets, braces), powers, radicals and fraction lines.

$4^2 \times (6 - 3)^2$ $= 4^2 \times 3^2$ $= 16 \times 9$ $= 144$	Calculate $6 - 3$ first as it is in the grouping symbols. Calculate 4^2 and 3^2 as they are powers & must be done before the multiplication Calculate 16×9 to get the answer.
$4 + 2 \times (6 - 3)^2$ $= 4 + 2 \times 3^2$ $= 16 + 2 \times 9$ $= 16 + 18$ $= 34$	Calculate $6 - 3$ first as it is in the grouping symbols. Calculate 3^2 as it is a power. Calculate 2×9 as it is multiplication and must be done before the addition. Calculate $16 + 18$ to get the answer.
$4 + (2 \times \{6 - 3\})^2$ $= 4 + (2 \times 3)^2$ $= 4 + 6^2$ $= 4 + 36$ $= 40$	Calculate $6 - 3$ first as it is in the inner grouping symbols. Calculate 2×3 as it is in the grouping symbols. Calculate 6^2 as it is a power and must be done before the addition. Calculate $4 + 36$ to get the answer.
$4 + 3 \times \sqrt{25}$ $= 4 + 3 \times 5$ $= 4 + 15$ $= 19$	Calculate $\sqrt{25}$ first as it is a square root, that is a radical or power. Calculate 3×5 as it is multiplication and must be done before the addition. Calculate $4 + 15$ to get the answer.
$\frac{6 + 3}{6 - 3}$ $= \frac{9}{3}$ $= 3$	Note: The fraction bar is a grouping symbol. i.e. $\frac{(6+3)}{(6-3)}$ Calculate the $6 + 3$ and the $6 - 3$ first as they are in the grouping symbols. Calculate $9 \div 3$ to get the answer.
$4 + 3 \times \sqrt{25 - 9}$ $= 4 + 3 \times \sqrt{16}$ $= 4 + 3 \times 4$ $= 4 + 12$ $= 16$	Calculate $\sqrt{25 - 9}$ first as it is in a square root which is a grouping symbol. Calculate $\sqrt{16}$ first as it is a square root which is a power. Calculate 3×4 as it is multiplication and must be done before the addition. Calculate $4 + 12$ to get the answer.

See if you can follow this one.

$$4 + \frac{10 + \sqrt{3^2 + 4^2}}{21 - 6 \times 3}$$

$$= 4 + \frac{10 + \sqrt{9 + 16}}{21 - 18}$$

$$= 4 + \frac{10 + \sqrt{25}}{3}$$

$$= 4 + \frac{10 + 5}{3}$$

$$= 4 + \frac{15}{3}$$

$$= 4 + 5$$

$$= 9$$