WorkBook

Long Division

a complete process

WorkNotes

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Teaching Long Division

OBJECT

To successfully teach the long division method showing full working.

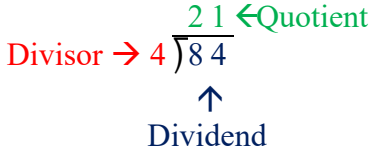
LANGUAGE

While I firmly believe that if you want "to do the maths you must be able to talk the maths" there are times when young learners do not need to know all the language. By all means use the words, however, a young learner should not be expected to know the words and the definitions are not required for learning division. That said to explain the process I will list the formal terms for division.

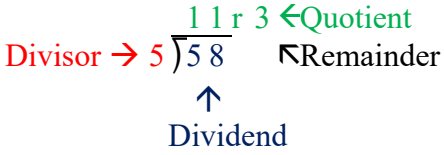
Divisor – the number you divide by
Dividend – the number being divided
Quotient – the answer when you divide
Remainder – the part left over when the dividend is not a multiple of the divisor.

However it is important to use and to be aware of the terms 'quotient' and 'remainder'.

Eg. 84 ÷ 4 = 21



Eg. 58 ÷ 5 = 11 r3



PROCESS

While people often incorrectly propagate the concept 'New mathematics', long division has never changed and I will fully demonstrate the method.

There is a method being taught that uses repeated subtraction to calculate answers for division. However, this is NOT long division. It is using alternate operations to avoid long division given that long division is often portrayed as being difficult.

METHOD

Simple division using long division method

The method can be summarised as;

D	**M**	**S**	**B**
i	u	u	r
v	l	b	i
i	t	t	n
d	i	r	g
e	p	a	
	y	c	d
		t	o
			w
			n

These steps will make sense as each example is calculated.

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Eg. 516 ÷ 3 =
This example of simple division, which can usually be calculated using mental processes, is to show the structure of long division.

Rewrite as a structural algorithm.

$$3\overline{)516}$$

As with all division you divide one number at a time from left to right.
First step. **D**ivide
The first digit of the dividend is 5. So divide 5 by 3. The 3 times tables gives us $3 \times 1 = 3$, $3 \times 2 = 6$, …
The correct choice is 1 as $3 \times 2 = 6$ is too large.

$$\begin{array}{r} 1 \\ 3\overline{)516} \end{array}$$

Second step. **M**ultiply
$3 \times 1 = 3$. This written vertically below the 5.

$$\begin{array}{r} 1 \\ 3\overline{)516} \\ 3 \end{array}$$

Third step. **S**ubtract
$5 - 3 = 2$. The 2 written vertically below the 5 and the 3.
The 2 is the remainder for the first division.

$$\begin{array}{r} 1 \\ 3\overline{)516} \\ \underline{3} \\ 2 \end{array}$$

Fourth step. **B**ring down
You now 'bring down' the next digit of the dividend. This gives you the next number to be divided by 3.

$$\begin{array}{r} 1 \\ 3\overline{)516} \\ \underline{3} \\ 21 \end{array}$$

Repeat the above steps
First step. **D**ivide
You now divide 21 by 3. The 3 times tables gives us $3 \times 7 = 3$ which is the correct choice.

$$\begin{array}{r} 17 \\ 3\overline{)516} \\ \underline{3} \\ 21 \end{array}$$

Second step. **M**ultiply
$3 \times 7 = 21$. This written vertically below the 21 so that each place value lines up.

$$\begin{array}{r} 17 \\ 3\overline{)516} \\ \underline{3} \\ 21 \\ 21 \end{array}$$

Third step. **S**ubtract
$21 - 21 = 0$. The 0 written vertically below the 1 (which is in the units column) since it is zero units.

$$\begin{array}{r} 17 \\ 3\overline{)516} \\ \underline{3} \\ 21 \\ \underline{21} \end{array}$$

$$\begin{array}{r} \\ \hline 0 \end{array}$$

Fourth step. **B**ring down
You now 'bring down' the next digit of the dividend. This gives you the next number to be divided by 3. Here it is 6. You will recall that 06 is 0 tens and 6 units, that is, it is 6

$$\begin{array}{r} 1\,7 \\ \hline 3\,)\overline{5\,1\,6} \\ 3 \\ \hline 2\,1 \\ 2\,1 \\ \hline 0\,6 \end{array}$$

Repeat the above steps
First step. **D**ivide
You now divide 6 by 3. The 3 times tables gives us $3 \times 2 = 6$ which is the correct choice.

$$\begin{array}{r} 1\,7\,2 \\ \hline 3\,)\overline{5\,1\,6} \\ 3 \\ \hline 2\,1 \\ 2\,1 \\ \hline 0\,6 \end{array}$$

Second step. **M**ultiply
$3 \times 2 = 6$. This written vertically below the 6 so that unit values line up.

$$\begin{array}{r} 1\,7\,2 \\ \hline 3\,)\overline{5\,1\,6} \\ 3 \\ \hline 2\,1 \\ 2\,1 \\ \hline 0\,6 \\ \quad 6 \end{array}$$

Third step. **S**ubtract
$6 - 6 = 0$. The 0 written vertically below the 6 (which is in the units column) since it is zero units.

$$\begin{array}{r} 1\,7\,2 \\ \hline 3\,)\overline{5\,1\,6} \\ 3 \\ \hline 2\,1 \\ 2\,1 \\ \hline 0\,6 \\ \quad 6 \\ \quad \overline{0} \end{array}$$

Fourth step. **B**ring down
There are no more digits to 'bring down' so the calculation is complete
Hence $516 \div 3 = 172$

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Estimation or Guess and Check

When doing long division we use one or more processes. These include approximation, rounding, estimation and/or ‘guess and check’. These are all valid and necessary mathematical methods.

Since usual that only multiplication tables for single digits are learned most divisions involve these numbers. So for numbers larger than 10 we have to round off the number to a multiple of 10 or 100 or 1000 etc. For example if you were dividing by 19 you would round up to 20 and use the 2 times tables to estimate the quotient.
When rounding the divisor the dividend should be rounded with zeros in the same place value. The long division process will determine if you have ‘guessed’ the correct quotient. It takes a bit of practise to ‘guess’ the best quotient as sometimes the rounding can be a significant portion of the divisor,
Other examples;

Quotient	Rounded off quotient	Times table used
32	30	3
47	50	5
68	70	7
124	100	1
237	200	2
188	200	2
309	300	3

Examples of rounded off division;

Division	Rounded off division	Times table	Division used	Closest times table result	First ‘guess’	
165 ÷ 32	160 ÷ 30	3	16 ÷ 3	$3 \times 5 = 15$	5	Good guess
224 ÷ 32	220 ÷ 30	3	22 ÷ 3	$3 \times 7 = 21$	7	Good guess
412 ÷ 68	410 ÷ 70	7	40 ÷ 7	$7 \times 6 = 42$	6	Good guess
1916 ÷ 237	1900 ÷ 200	2	1900 ÷ 2	$2 \times 9 = 18$	9	Bad guess
1248 ÷ 178	1200 ÷ 200	2	1200 ÷ 2	$2 \times 6 = 18$	6	Bad guess

Because some guesses may be wrong I would recommend that the working out be done using a pencil so you can erase the incorrect guess and proceed with the correct guess. Examples of this process will be given below.

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Eg. 612 ÷ 17 =
For this division by 17 round off to 20 and then use the two times tables.

Rewrite as a structural algorithm.

$$17\overline{)612}$$

As with all division you divide one number at a time from left to right.
First step. **D**ivide
The first digit of the dividend is 6 which is smaller than 17. So use the first two digits. So divide 61 by 17. The guess will be 60 ÷ 20. The 2 times tables gives us 2 × 3 = 6.
3 has unit place value it must line vertically with the units place value of 61. So is it must be above the '1'.

$$\begin{array}{r} 3 \\ 17\overline{)612} \end{array}$$

Second step. **M**ultiply
3 × 17 = 51. This written vertically below the 61.

$$\begin{array}{r} 3 \\ 17\overline{)612} \\ 51 \end{array}$$

Third step. **S**ubtract
61 – 51 = 10. The 10 written vertically below the 51.
The 10 is the remainder for the first division.

$$\begin{array}{r} 3 \\ 17\overline{)612} \\ \underline{51} \\ 10 \end{array}$$

Fourth step. **B**ring down
You now 'bring down' the next digit of the dividend. This gives you the next number to be divided by 17.

$$\begin{array}{r} 3 \\ 17\overline{)612} \\ \underline{51} \\ 102 \end{array}$$

Repeat the above steps
First step. **D**ivide
The first 2 digits of 102 is 10 which is smaller than 17. So use all three digits. So divide 102 by 17. The guess will be 100 ÷ 20. The 2 times tables gives us 2 × 5 = 10.
2 was the number brought down so 5 must line vertically with the '2'

$$\begin{array}{r} 35 \\ 17\overline{)612} \\ \underline{51} \\ 102 \end{array}$$

Second step. **M**ultiply
5 × 17 = 85. This written vertically below the 102 so that each place value lines up.

$$\begin{array}{r} 35 \\ 17\overline{)612} \\ \underline{51} \\ 102 \\ 85 \end{array}$$

Third step. **S**ubtract
21 – 21 = 0. The 0 written vertically below the 1 (which is in the units column) since it is zero units.

$$\begin{array}{r} 35 \\ 17\overline{)612} \\ 51 \\ \hline 102 \\ 85 \\ \hline 17 \end{array}$$

Here we see a problem because the remainder is equal to the divisor and so the guess was too small. So we go back to step 2 with our new guess of 6,
Second step (2nd attempt). **M**ultiply
5 × 17 = 85. This written vertically below the 102 so that each place value lines up.

$$\begin{array}{r} 36 \\ 17\overline{)612} \\ 51 \\ \hline 102 \\ 102 \end{array}$$

Third step. **S**ubtract
21 – 21 = 0. The 0 written vertically below the 1 (which is in the units column) since it is zero units.

$$\begin{array}{r} 36 \\ 17\overline{)612} \\ 51 \\ \hline 102 \\ 102 \\ \hline 0 \end{array}$$

Fourth step. **B**ring down
There are no more digits to 'bring down' so the calculation is complete
Hence 612 ÷ 17 = 36

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Eg. 4864 ÷ 43 =
For this division by 17 round off to 20 and then use the two times tables.

Rewrite as a structural algorithm.

$$43\overline{)4\,8\,6\,4}$$

As with all division you divide one number at a time from left to right.
First step. **D**ivide
The first digit of the dividend is 4 which is smaller than 43. So use the first two digits. So divide 48 by 43. The guess will be 40 ÷ 40 which is 1.
3 has unit place value it must line vertically with the units place value of 61. So is it must be above the '1'.

$$\begin{array}{r} 1 \\ 43\overline{)4\,8\,6\,4} \end{array}$$

Second step. **M**ultiply
1 × 43 = 43. This written vertically below the 5.

$$\begin{array}{r} 1 \\ 43\overline{)4\,8\,6\,4} \\ 4\,3 \end{array}$$

Third step. **S**ubtract
5 – 3 = 2. The 2 written vertically below the 5 and the 3.
The 2 is the remainder for the first division.

$$\begin{array}{r} 1 \\ 43\overline{)4\,8\,6\,4} \\ \underline{4\,3} \\ 5 \end{array}$$

Fourth step. **B**ring down
You now 'bring down' the next digit of the dividend. This gives you the next number to be divided by 43.

$$\begin{array}{r} 1 \\ 43\overline{)4\,8\,6\,4} \\ \underline{4\,3} \\ 5\,6 \end{array}$$

Repeat the above steps
First step. **D**ivide
The first 2 digits of 102 is 10 which is smaller than 17. So use all three digits. So divide 102 by 17. The guess will be 100 ÷ 20. The 2 times tables gives us 2 × 5 = 10.
2 was the number brought down so 5 must line vertically with the '2'

$$\begin{array}{r} 1 \\ 43\overline{)4\,8\,6\,4} \\ \underline{4\,3} \\ 5\,6 \end{array}$$

Second step. **M**ultiply
5 × 17 = 85. This written vertically below the 102 so that each place value lines up.

$$\begin{array}{r} 3\,5 \\ 17\overline{)6\,1\,2} \\ \underline{5\,1} \\ 1\,0\,2 \\ 8\,5 \end{array}$$

Third step. **S**ubtract
21 – 21 = 0. The 0 written vertically below the 1 (which is in the units column) since it is zero units.

$$\begin{array}{r} 35 \\ 17\overline{)612} \\ 51 \\ \hline 102 \\ 85 \\ \hline 17 \end{array}$$

Here we see a problem because the remainder is equal to the divisor and so the guess was too small. So we go back to step 2 with our new guess of 6,
Second step (2nd attempt). **M**ultiply
5 × 17 = 85. This written vertically below the 102 so that each place value lines up.

$$\begin{array}{r} 36 \\ 17\overline{)612} \\ 51 \\ \hline 102 \\ 102 \end{array}$$

Third step. **S**ubtract
21 – 21 = 0. The 0 written vertically below the 1 (which is in the units column) since it is zero units.

$$\begin{array}{r} 36 \\ 17\overline{)612} \\ 51 \\ \hline 102 \\ 102 \\ \hline 0 \end{array}$$

Fourth step. **B**ring down
There are no more digits to 'bring down' so the calculation is complete
Hence 612 ÷ 17 = 36