

# **WorkBook**

## **GEOMETRY**

### **Definitions and Angle Properties**

## **WorkNotes**

# WorkBook

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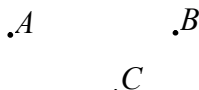
# WorkBook

## GEOMETRY

### Point

- A point is a position in space. A point has no dimensions (no size).
- A Point is denoted by an upper case letter.
- Represented by a dot.

Eg

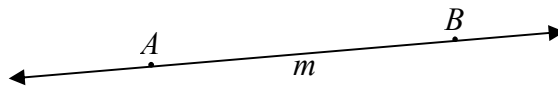


The points  $A$ ,  $B$ , and  $C$

### Line

- A line is the shortest path between 2 points. A line has one dimension (length).
- A line is denoted by two of the points it passes through or by a lower case letter.
- A line that drawn is only a representation of the line.

Eg.



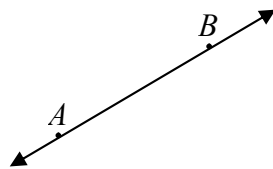
The line  $m$ , or the line  $AB$ .

### Types of lines

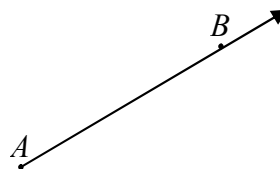
In common usage, they are all called lines.

- **Line** - Infinite in both directions.
- **Ray** - Starting at a point, and infinite in one direction. The end point is usually used in naming the ray.
- **Interval** - Joins two points. An interval is finite in length.

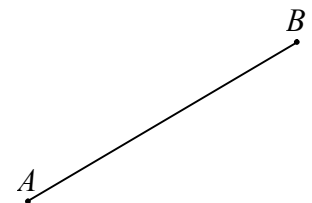
Eg.



The line  $AB$ .



The ray  $AB$ .



The interval  $AB$ .

### Plane

- A plane has two dimensions (length and width). A plane has no thickness. A plane is usually thought of as a flat surface.
- A plane is denoted by three or more points it passes through.
- A plane can also be defined by a pair of lines.

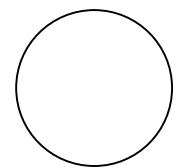
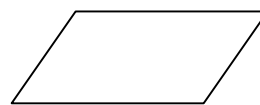
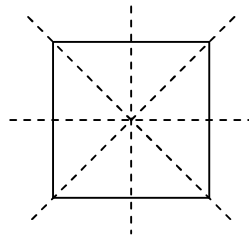
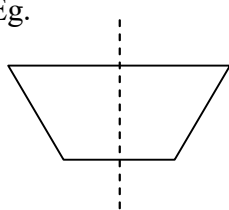
### Space

- Space has three dimensions (length, width and height).
- Space can be defined by a pair of planes.

### Axis of Symmetry

- A line though a shape or object such that one side of the line is a reflection of the other side.

Eg.



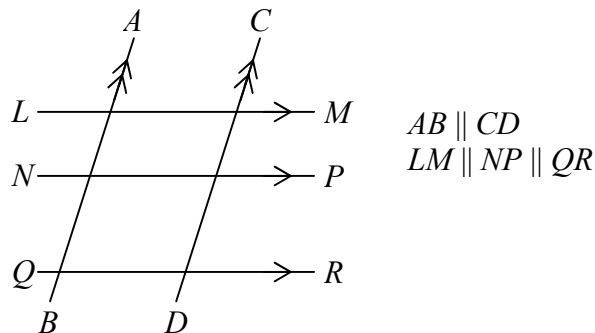
1 axis of symmetry    4 axes of symmetry    0 axis of symmetry     $\infty$  axes of symmetry

# WorkBook

## Parallel lines

- Parallel lines are lines that remain a constant distance apart.
- Indicated lines are parallel by (matching) arrows on the lines.
- The symbol used to indicate parallelism is  $\parallel$ .

Eg.



A line crossing two or more parallel line is called a **transversal**.

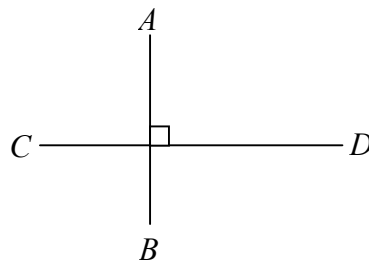
Eg.

$AB$  is a transversal of line  $LM$  and  $NP$ .

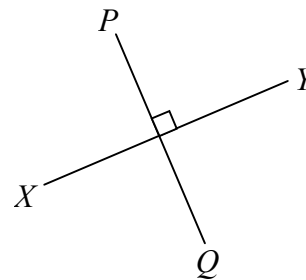
## Perpendicular Lines

- Perpendicular lines are at right angles to each other.
- The symbol used to indicate perpendicular is  $\perp$ .

Eg.



$AB \perp CD$

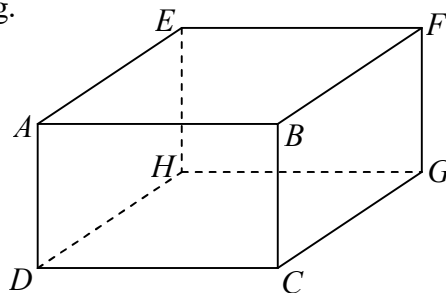


$PQ \perp XY$

## Skew Lines

- Skew lines are non-parallel lines that do not intersect.
- Skew lines are on different planes.

Eg.



$AB$  and  $FG$  are skew lines.  
 $AB$  and  $CG$  are skew lines.

**Note:** In the above diagram;

$AB$  is on plane  $ABCD$ . Hence  $AB$  and  $FG$  are skew lines because they are on different planes.  
 $FG$  is on plane  $EFGH$ .

$CG$  is on plane  $BCGF$ . Plane  $BCGF$  and plane  $DCGH$  intersect at the line  $CG$ .  
 $CG$  is on plane  $DCGH$ .

Plane  $ABCD \perp$  plane  $BCGF$

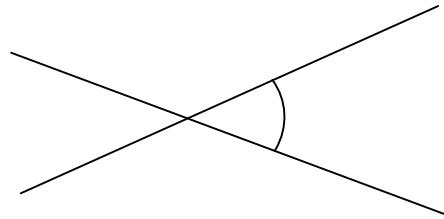
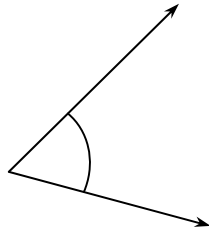
Plane  $ABCD \parallel$  plane  $EFGH$

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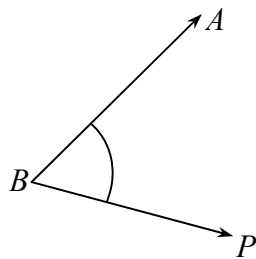
## Angles

- Angles are formed by the rotation of a line.
- Angles are formed by the intersection of two lines.
- The point of rotation, or intersection, is called the vertex of the angle.

Eg.



- Angles are denoted by;
- their vertex.
  - two points on line, including the intersecting point.



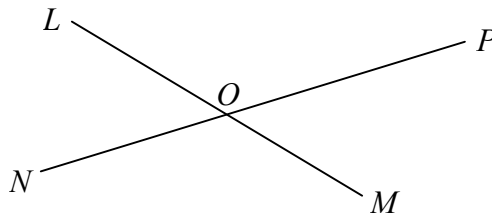
$\hat{B}$  or  $\angle B$   
 $\hat{ABP}$  or  $\angle ABP$

Both symbols indicate angle.

### Notes:

- In some diagrams, there may be several angles that could be called the same if only the vertex is used.

Eg. There are four angles that may be called  $\hat{O}$  or  $\angle O$  in the diagram below.



However, there is only one angle called  $\hat{LOP}$  or  $\angle LOP$ . Therefore, you must use three points to name these angles.

- When using three points to name an angle, the **vertex must be the centre point** in that name.

$B$

## Measuring Angles

Angles are measured in degree, where, one degree is  $\frac{1}{360}$  of a revolution.

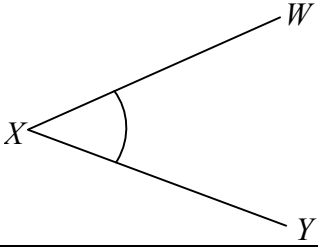
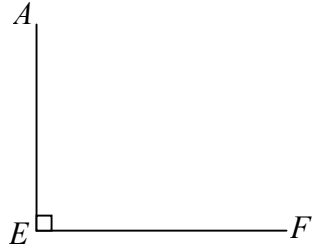
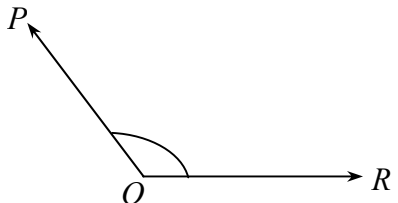
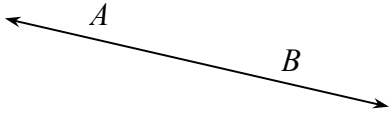
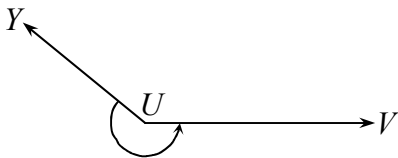
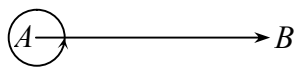
$$1^\circ \text{ (degree)} = 60' \text{ (minutes)}$$

$$1' \text{ (minutes)} = 60'' \text{ (seconds)}$$

A protractor is a geometrical instrument used for measuring angles. For notes on the use of a protractor, see attachment - **USING A PROTRACTOR - MEASURING ANGLES**

# WorkBook

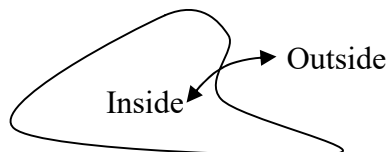
## Types of Angles

Type	Size	Example
<b>Acute</b>	$0^\circ < \theta < 90^\circ$	
<b>Right</b>	$90^\circ$	
<b>Obtuse</b>	$90^\circ < \theta < 180^\circ$	
<b>Straight</b>	$180^\circ$	
<b>Reflex</b>	$180^\circ < \theta < 360^\circ$	
<b>Revolution</b>	$360^\circ$	

- Planar shapes**
- Planar shapes are 2 dimensional shapes
  - Shapes formed on a plane

- Closed Curve**
- A closed curve is a curve such that to get from the “inside” to the “outside”, you must cross the curve.
  - A closed curve is a planar shape.

Eg.

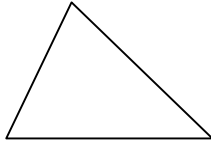
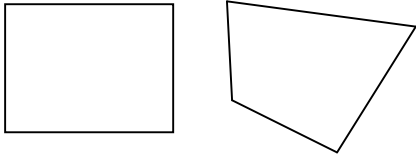
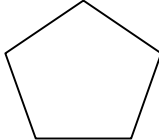
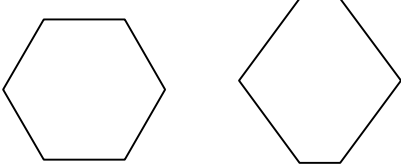
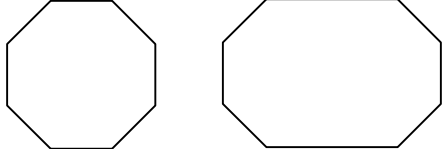
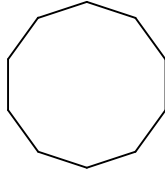


- Polygon**
- A polygon is a closed curve with straight sides.
- Note:** A line is a straight curve.

## WorkBook

### Types of Polygons

- The common types of polygons are listed below.

Types of Polygons	No. Of Sides	Diagram
<b>Triangle</b>	3	
<b>Quadrilateral</b>	4	
<b>Pentagon</b>	5	
<b>Hexagon</b>	6	
<b>Octagon</b>	8	
<b>Decagon</b>	10	

### Triangles

Triangles are classified using their angles or sides.

### Types of triangles and properties of Triangles

#### Classification by angles.

Triangle can be grouped using the lengths of the sides of the triangle

**Acute triangles** have all acute angles.

**Right triangles** have a right angle.

**Obtuse triangles** have one obtuse angle.

#### Classification by sides.

Triangle can be grouped using the size of the angles of the triangle

**Scalene triangles** have all three sides of different length.

**Isosceles triangles** have two equal sides (therefore two equal angles).

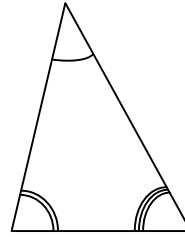
**Equilateral triangles** have three equal sides (therefore three equal angles).

Some triangles can be labelled using a combination of the two groups of triangles.  
Eg. right isosceles triangle, obtuse isosceles triangle, ...

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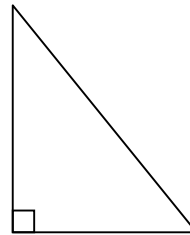
## Acute triangles

- Acute triangles have all acute angles.



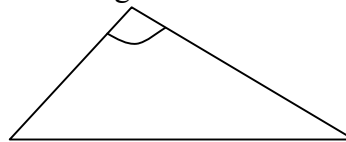
## Right triangles

- Right triangles have a right angle.



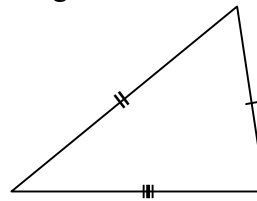
## Obtuse triangles

- Obtuse triangles have one obtuse angle.



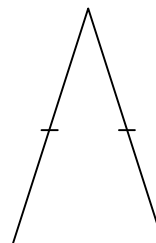
## Scalene triangles

- have all three sides of different length



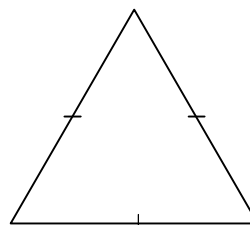
## Isosceles triangles

- Isosceles triangles have two equal sides.
- Isosceles triangles have two equal angles.
- The perpendicular bisector of the base of an isosceles triangle passes through the opposite vertex.



## Equilateral triangles

- Equilateral triangles have three equal sides.
- Equilateral triangles have three equal angles (all  $60^\circ$ ).
- The perpendicular bisector of the sides of an equilateral triangle passes through the opposite vertex.

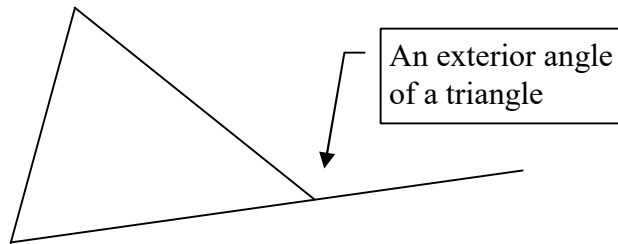




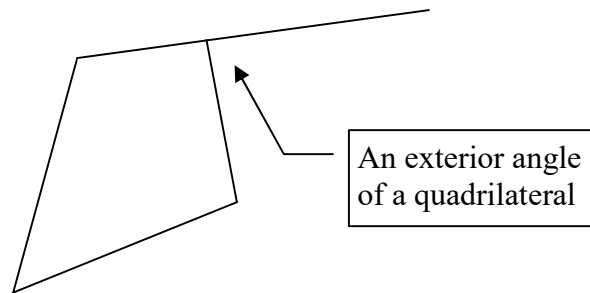
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**Exterior angles** An exterior angle is formed when a side of a polygon is extended. There can be several exterior angles on a polygon.

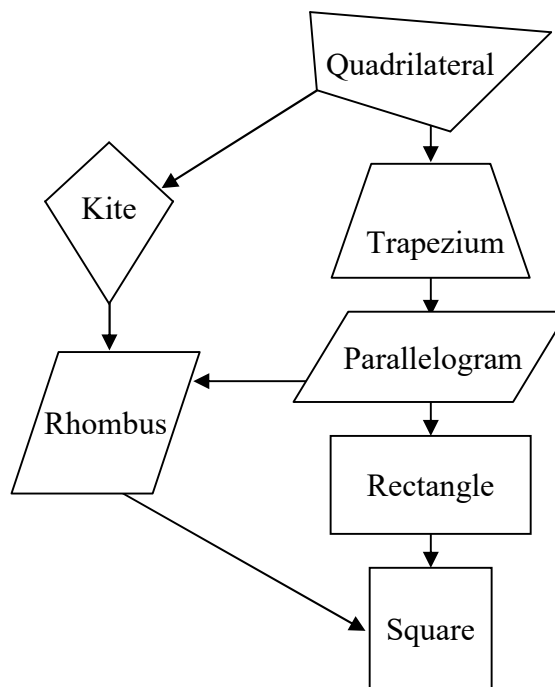
Example



Example



**Quadrilaterals** Quadrilaterals are classified using their sides and/or the parallelism of their sides. The types of quadrilateral can be linked as in the diagram below.



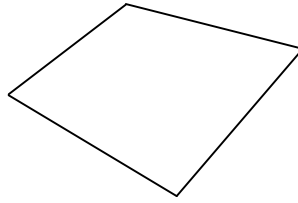
As you move from the top to the bottom, the quadrilaterals have an increasing number of properties.

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### Properties of Special Quadrilaterals

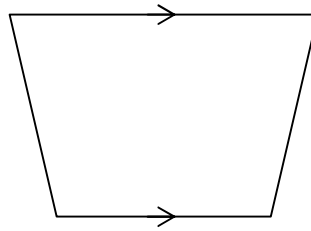
#### Quadrilateral

- Four sides.

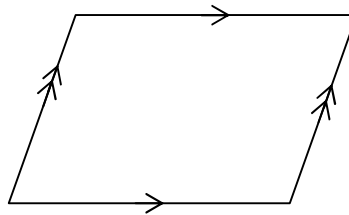


#### Trapezium (one pair of parallel sides)

- Four sides.
- One pair of parallel sides

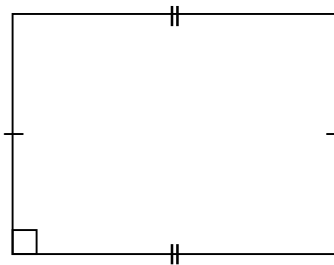


#### Parallelogram (opposite sides are parallel)



- Four sides.
- Opposite sides are parallel.
- Opposite sides are equal.
- Opposite angles are equal.
- Diagonals bisect.

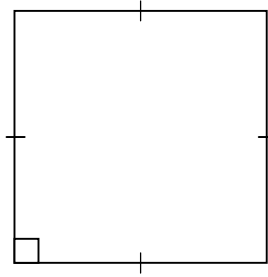
#### Rectangle (opposite sides are equal, all angles are $90^\circ$ )



- Four sides.
- Opposite sides are parallel.
- Opposite sides are equal.
- All angles are  $90^\circ$ .
- Diagonals bisect.
- Diagonals are equal.
- There are two axes of symmetry.

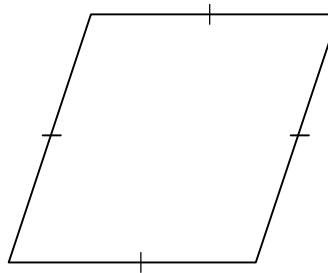
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**Square** (all sides are equal, all angles are  $90^\circ$ ).



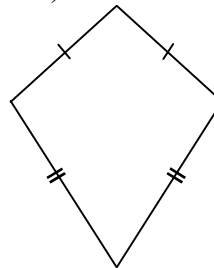
- Four sides.
- Opposite sides are parallel.
- All sides are equal.
- All angles are  $90^\circ$ .
- Diagonals bisect.
- Diagonals are perpendicular (i.e. they are perpendicular bisectors).
- Diagonals are equal.
- There are four axes of symmetry.

**Rhombus** (all sides equal)



- Four sides.
- Opposite sides are parallel.
- All sides are equal.
- Opposite angles are equal.
- Diagonals bisect.
- Diagonals are perpendicular (i.e. they are perpendicular bisectors).
- There are two axes of symmetry.

**Kite** (two pairs of equal adjacent sides)



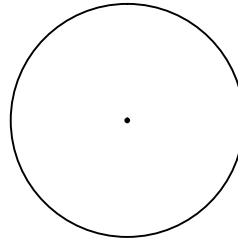
- Two pairs of equal adjacent sides.
- Diagonals are perpendicular.
- There is one axis of symmetry.

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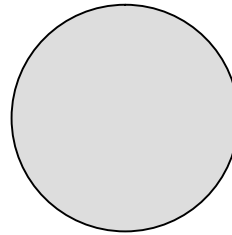
## Circles

There are two definitions of a circle, both are important

- A circle is the set of points, on a plane, equidistant from a fixed point (the centre). A circle is the path satisfying this condition, it is a curve.
- In measurement, this is referred to as the **circumference**.



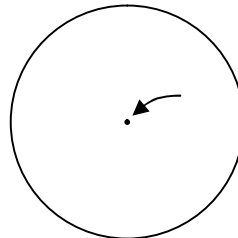
- A circle is a planar shape, a set of points lying within the circle defined above. A circle is a closed curve with infinite axes of symmetry.



## Parts of a circle

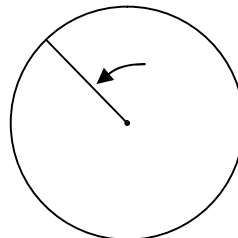
### Centre

- The point (inside the circle) that is equidistant from all points on the circle.
- The fixed point defining the circle
- Radii is the plural



### Radius

- The line from the center of the circle to the circumference (circumference)
- The fixed distance defining the circle (circumference)
- The distance from the center of the circle to the circumference

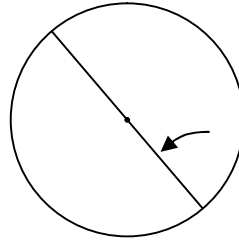


## Parts of a circle (Cont.)

### Diameter

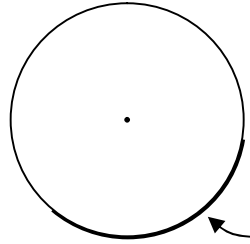
- A line passing through the center of a circle that joins two points on the circle.
- A line passing through the circle that bisects the circle.

## WorkBook



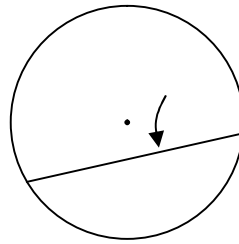
### Arc

- a continuous portion of a circle (or curve)
- part of the circumference of a circle (or curve).



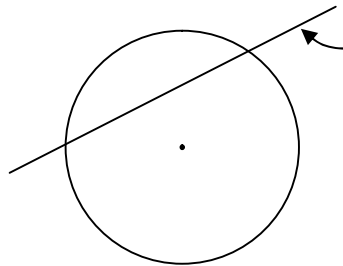
### Chord

- a line joining two points on a circle (or curve)



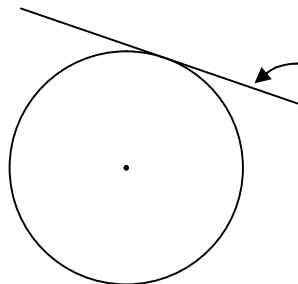
### Secant

- a line passing through two points on a circle (or curve)



### Tangent

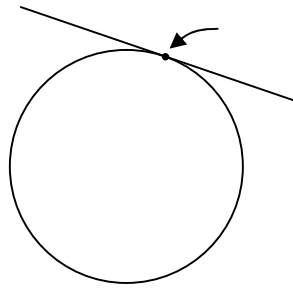
- a line that touches a circle (or curve)
- a line that intersects a circle (or curve) only once



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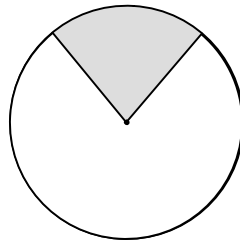
## Point of contact

- the point where the tangent intersects the circle (or curve)

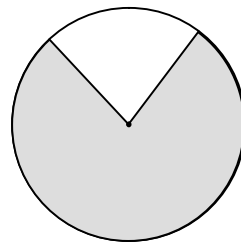


## Sector

- part of a circle formed by a pair of radii



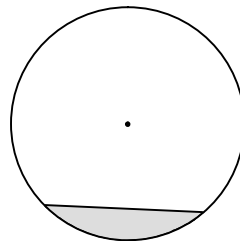
Minor sector



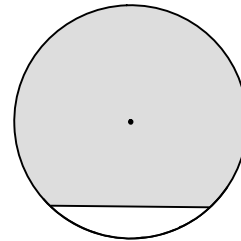
Major sector

## Segment

- part of a circle formed by a chord or secant



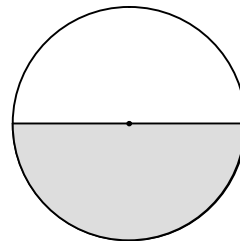
Minor segment



Major segment

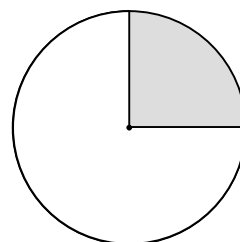
## Semi Circle

- part of a circle formed by a diameter
- half a circle



## Quadrant

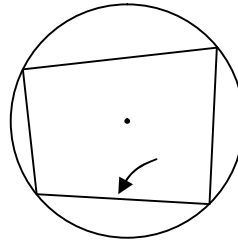
- quarter of a circle formed by a pair of radii



## WorkBook

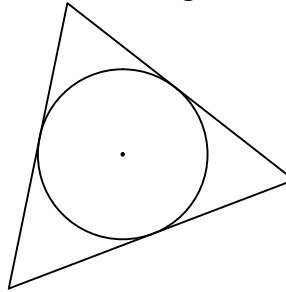
### Cyclic polygons

- the smallest circle which contains the polygon completely within it.
- the circle touches each vertex of the polygon
- the circle is called a **circumcircle**.
- The centre of a cyclic polygon is known as the **circumcentre**.



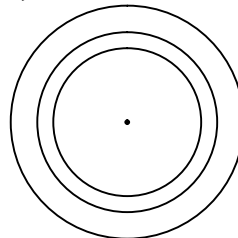
### In-circle and in-centre

- The in-circle or inscribed circle of a triangle is the largest circle contained in the triangle.
- The in-circle touches (is tangent to) the three sides.
- The centre of the in-circle is called the triangle's in-centre



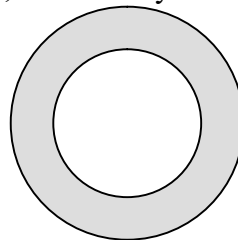
### Concentric circles

- circles that share the same centre, but have different radii



### Annulus

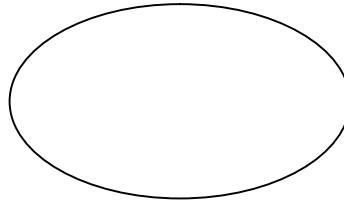
- The space, or portion of a plane, bounded by two concentric circles



# WorkBook

## Ellipse

- Closed curve
- The set of points in a plane such that the sum of the distances to two fixed points is a constant.
- Often thought of as a flattened circle. Sometimes referred to as an oval.



## Solids

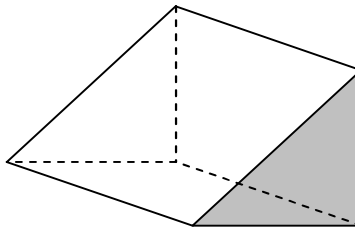
- solids are 3 dimensional shapes
- solids are shapes formed in space
- solids can either be categorised or are irregular

## Types of Solids

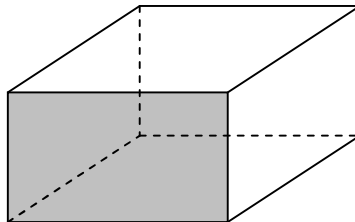
### Prisms

- Prisms are solids that have a congruent cross-section.
- All faces of a prism are planar
- All faces of a prism other than the two congruent cross-section faces are rectangles.
- Prisms are named according to the shape of the base.

### Triangular prism

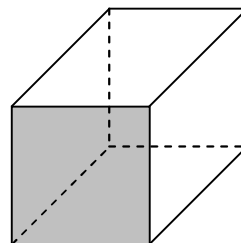


### Rectangular prism



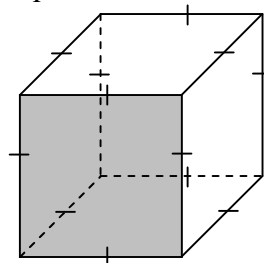
### Square prism

All faces of a square prism are congruent rectangles except the cross-section squares.



### Cube

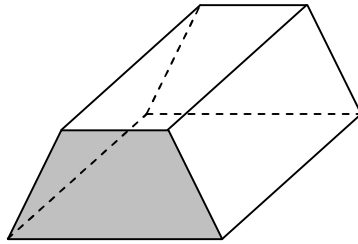
All faces of a cube are congruent squares.



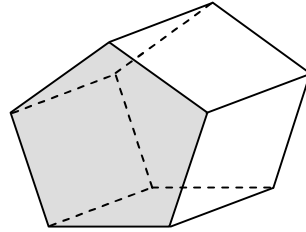


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### Trapezoidal prism



### Pentagonal prism



There are many other prisms.

## Pyramids

- Pyramids are solids that have all their faces, other than the base, meeting at a point (apex).
- All faces of a pyramid are planar
- All faces of a pyramid other than the base are triangles.
- Pyramids are named according to the shape of the base.

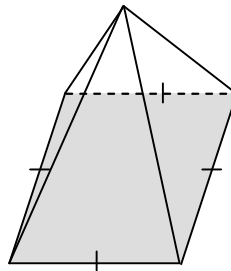
### Triangular pyramid

#### Tetrahedron

All faces are congruent equilateral triangles

### Rectangular pyramid

### Square pyramid



There are many other pyramids

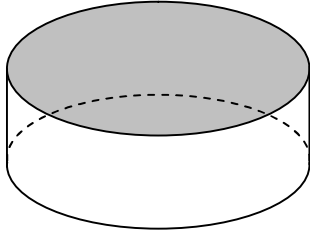
## WorkBook

### Spheres

There are two definitions of a sphere, both are important

- A sphere is the set of points, in space, equidistant from a fixed point (the centre). A sphere is the curved surface satisfying this condition.
- A sphere is a solid defined by the sphere above.

### Other regular solids



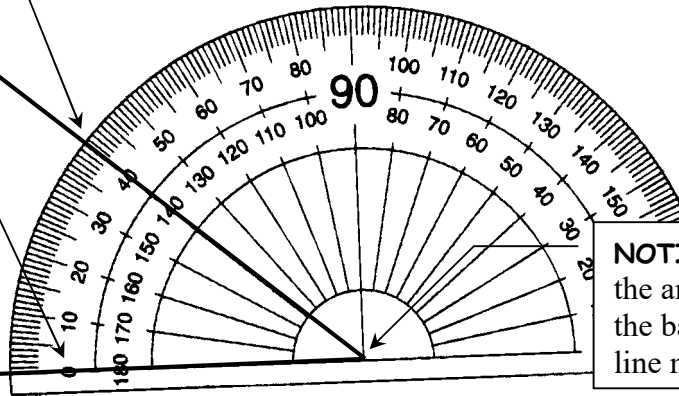
# WorkBook

## USING A PROTRACTOR - MEASURING ANGLES

**Example 1** – An acute angle of  $40^\circ$

**NOTICE** that the second ray of the angle lines up with  $40^\circ$  on the outside scale.

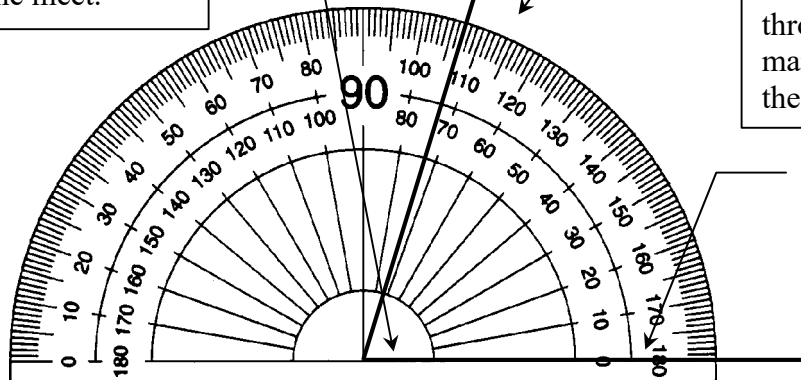
**NOTICE** that one ray of the angle lines up with  $0^\circ$  on the outside scale.



**NOTICE** that the vertex of the angle is exactly where the base line and the  $90^\circ$  line meet.

**Example 1** – An acute angle of  $73^\circ$

**NOTICE** that the vertex of the angle is exactly where the base line and the  $90^\circ$  line meet.



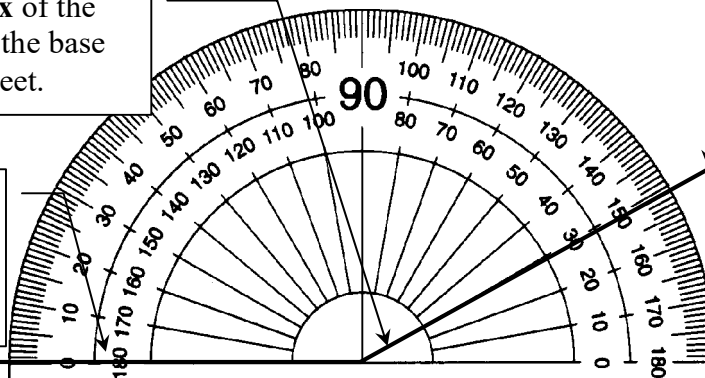
**NOTICE** that the second ray of the angle lines up with  $73^\circ$  on the inside scale. The arm passes through the mark that is 3 marks away from zero, past the  $70^\circ$  (also  $110^\circ$ )

**NOTICE** that one ray of the angle lines up with  $0^\circ$  on the inside scale.

**Example 1** – An obtuse angle of  $151^\circ$

**NOTICE** that the vertex of the angle is exactly where the base line and the  $90^\circ$  line meet.

**NOTICE** that one ray of the angle lines up with  $0^\circ$  on the outside scale.



**NOTICE** that the second ray of the angle lines up with  $151^\circ$  on the outside scale. The arm passes through the mark that is 1 mark away from zero, past the  $150^\circ$