WorkBook

FACTORISING QUADRATIC TRINOMIALS

WorkNotes

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FACTORISATION

Common Factors

Common factors are numbers or pronumerals that can be wholly divided into each part of an expression.

Example 1

Solution	Explanations
$6d + 8$	Both 6 and 8 are divisible by 2. $$6 \div 2 = 3 \text{ and } 8 \div 2 = 4$$
$= 2(3d + 4)$	The common factor, in this case 2, is written in front of the brackets and the quotients are written inside the brackets. $$6d \div 2 = 3d \text{ and } 8 \div 2 = 4$$

Example 2

Solution	Explanations
$4m^2 - 5m$	Both m^2 and m are divisible by m. $$m^2 \div m = m \text{ and } m \div m = 1$$
$= m(4m - 5)$	The common factor, in this case m, is written in front of the brackets and the quotients are written inside the brackets. $$4m^2 \div 4m = m \text{ and } 5m \div m = 5$$

Example 3

Solution	Explanations
$6a^2 - 8ad$	Both $6a^2$ and $8ad$ are divisible by 2 and a. $$a^2 \div a = a \text{ and } a \div a = 1$$ $$6 \div 2 = 3 \text{ and } 8 \div 2 = 4$$
$= 2a(3a - 4d)$	The common factor, in this case $2a$, is written in front of the brackets and the quotients are written inside the brackets. $$6a^2 \div 2a = 3a \text{ and } 8ad \div 2a = 4d$$

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Grouping

Grouping is used to factorise expressions with 4 or more terms.

Example 1

Solution	Explanations
$ac + ad + bc + bd$	The expression is separated into two parts. In most cases this will not require re-arranging. $ac + ad \,/\, + bc + bd$
$= a(c + d) + b(c + d)$	Each part of the expression is factorised, removing common factors. (*See above*)
$= (c + d)(a + b)$	$a\underline{(c + d)} + b\underline{(c + d)}$ It can be seen that $(c + d)$ is a common factor. Remove this common factor.

Example 2

Solution	Explanations
$15t^3 – 9t^2 + 25t – 15$	The expression is separated into two parts. In most cases this will not require re-arranging. $15t^3 – 9t^2 \,/\, + 25t – 15$
$= 3t^2(5t – 3) + 5(5t – 3)$	Each part of the expression is factorised, removing common factors. (*See above*)
$= (5t – 3)(3t^2 + 5)$	$3t^2\underline{(5t – 3)} + 5\underline{(5t – 3)}$ It can be seen that $(5t – 3)$ is a common factor. Remove this common factor.

Quadratic Expressions

Quadratic Factorisation		
ALWAYS remove **Common Factors First**		
2 Terms	Difference of 2 Squares	$4u^2 - 49$
3 Terms	Monic Trinomial	$b^2 - 21b + 108$
	Non-Monic Trinomial	$40v^2 - 71v + 21$
4 Terms (or More)	Grouping	$15t^3 – 9t^2 + 25t - 15$

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Difference of 2 squares

A Difference of Two Squares will factorise to a product of conjugates.
Conjugates are binomial expressions that only by the sign separating the two terms in the binomial expressions.

Eg. $(u - 8)$ *and* $(u + 8)$ are conjugates
$(8u - k)$ and $(8u + k)$ are conjugates

Example 1

Solution	Explanations
$u^2 - 64$	The two terms separated by the minus sign are both squares. $u^2 = (u)^2$ and $64 = (8)^2$ i.e. $u^2 - 64 = (u)^2 - (8)^2$
$= (u - 8)(u + 8)$	Each bracket contains the square roots, one bracket has a negative sign and the other bracket has a positive sign.

Example 2

Solution	Explanations
$25k^2 - 36v^2$	The two terms separated by the minus sign are both squares. $k^2 = (k)^2$ and $25 = (5)^2$ $v^2 = (v)^2$ and $36 = (6)^2$ i.e. $25k^2 - 36v^2 = (5)^2(k)^2 - (6)^2(v)^2$
$= (5k - 6v)(5k + 6v)$	Each bracket contains the square roots, one bracket has a negative sign and the other bracket has a positive sign.

Perfect Squares

Perfect squares result from the squaring of a binomial expression.

Eg. $(t + 5)^2 = t^2 + 10t + 25$

Example 1

Solution	Explanations
$h^2 + 16h + 64$	To be a perfect square the expression must contain two squared terms, and the remaining term should be double the product of the two square roots. $h^2 = (h)^2$ and $64 = (8)^2$ and $2 \times h \times 8 = 16h$
$= (h + 8)^2$	The solution is a bracket that contains the two square roots separated by the same sign as the middle sign of the trinomial.

Example 2

Solution	Explanations
$16a^2 - 40as + 25s^2$	To be a perfect square the expression must contain two squared terms, and the remaining term should be double the product of the two square roots. $16 = (4)^2$ and $a^2 = (a)^2$ $25 = (5)^2$ and $s^2 = (s)^2$ and $2 \times 4a \times 5s = 40as$ *i.e.* $(4a)^2 - (2 \times 4a \times 5s) + (5s)^2$
$= (4a - 5s)^2$	The solution is a bracket that contains the two square roots separated by the same sign as the middle sign of the trinomial.

Quadratic trinomials

Quadratic trinomials are of the form $ax^2 + bx + c$ where a, b, and c are real numbers and $a > 0$. Two special cases, Difference of 2 Squares (which are quadratic binomials) and Perfect Squares (which are quadratic trinomials), have special factorisation methods as outlined above.

Quadratic trinomials are either monic or non-monic. They are factorised differently.

Note:

Monic means one	$b^2 - 21b + 108$	(the coefficient of b^2 is 1)
Non-monic means "not one"	$40k^2 - 57k + 20j^2$	(the coefficient of k^2 is 40)

Monic

Example 1

Solution	Explanations
$b^2 - 21b + 108$	If the trinomial factorises, each bracket will contain a *b*.
$= (b \quad)(b \quad)$	The constant is +108 (the + is important) What are the two numbers that multiply to give 108 and add to 21? (don't worry about the –21 yet) + 108 9 12 Write two brackets containing this information.
$= (b \quad 9)(b \quad 12)$	Because 108 is positive, the two numbers have the same sign. Because the middle term is negative, both numbers are negative. + 108 -9 -12
$= (b - 9)(b - 12)$	Insert the signs.

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Example 2

Solution	Explanations
$m^2 - 3m - 40$	If the trinomial factorises, each bracket will contain an *m*.
$= (m \quad)(m \quad)$	The constant is - 40 (the - is important) What are the two numbers that multiply to give 40 and subtract to 3? (don't worry about the –3 yet) - 40 5 8 Write two brackets containing this information.
$= (m \quad 5)(m \quad 8)$	Because 40 is negative, the two numbers have different signs. Because the middle term is negative, the 'bigger' number is negative. - 40 +5 -8 By 'bigger', we refer to the magnitude or size of the number. However, remember 5 > -8.
$= (m + 5)(m - 8)$	Insert the signs.

Non-Monic

Example 1

Solution	Explanations
$5x^2 + 11x + 6$	Multiply the leading co-efficient and the constant. i.e. $5 \times 6 = +30$. (the + is important)
	What are the two numbers that multiply to give 30 and add to 30? (don't worry about the +11 yet) + 30 5 6
$= 5x^2 + 5x + 6x + 6$	Because 30 is positive, the two numbers have the same sign. Because the middle term is positive, the two numbers are positive. + 30 +5 +6 Rewrite the expression, separating the middle term into two parts, using these factors.
$= 5x(x + 1) + 6(x + 1)$	Use grouping to factorise this 4 term expression
$= (x + 1)(5x + 6)$	

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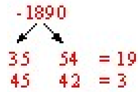
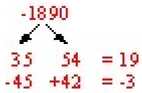
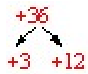
Example 2

Solution	Explanations
$54g^2 - 3g - 35$	Multiply the leading co-efficient and the constant. i.e. $54 \times -35 = -1890$. (the - is important)
	What are the two numbers that multiply to give 1890 and subtract to 3? (don't worry about the -3 yet) 
$= 54g^2 - 45g + 42g - 35$	Because 1890 is negative, the two numbers have different signs. Because the middle term is negative, the 'bigger' number is negative.  By 'bigger', we refer to the magnitude or size of the number. However, remember 42 > -45. Rewrite the expression, separating the middle term into two parts, using these factors.
$= 9g(6g - 5) + 7(6g - 5)$	Use grouping to factorise this 4 term expression
$= (6g - 5)(9g + 7)$	

Mutiple Steps

Some expressions can be factorised using more then one of the above methods. Remember to always look for common factors first.

Example 1

Solution	Explanations
$2m^2 + 30m + 72$	All terms are divisible by 2. (2 is a common factor.
$2(m^2 + 15m + 36)$	The bracket contains a monic trinomial. If the trinomial factorises, each bracket will contain an *m*.
$= 2(m \quad)(m \quad)$	The constant is +36 (the + is important) What are the two numbers that multiply to give 36 and add to 15? (don't worry about the +15 yet)  Write two brackets containing this information.
$= 2(m \quad 3)(m \quad 12)$	Because 36 is positive, the two numbers have the same sign. Because the middle term is positive, both numbers are positive. 
$= 2(m + 3)(m + 12)$	Insert the signs.